4.2 Filling the Boxes: Finding Volume

Focus Question What is a strategy for finding the volume of a rectangular prism? Explain why the strategy works.

Launch

Ask the students what they remember about computing volumes of rectangular prisms.

Hold up one of the boxes from Problem 4.1, Question B.

- *What is the volume of this box?*

During Problem 4.1, Question B, part (4), students found the volume informally by counting the cubes needed to fill the box. Revisiting that question when launching Problem 4.2 formalizes the concept of volume.

- *How could I find the volume? Is there another way?*

If students do not have a formula, they should be able to recall it as they work on Question A.

In this Problem, two students were asked to find the volume of boxes made from nets. Each student used a different strategy.

Briefly describe the two methods. Be sure not to give students the formulas associated with the methods, as students will develop the formulas during the Problem.

Your challenge is to figure out which method is correct, and then use the strategy to find volumes of boxes.

Key Vocabulary

- volume

Materials

Accessibility Labsheet

- 4.2: Volume Table

- centimeter cubes
- nets from Labsheet 4.1B (optional)
- Virtual Box Activity
- 3D Geometry Tool

Explore

Question A should give you a chance to observe the strategies students are using to find volume.

- *What strategies are you using to find the volume?*

If students say counting, ask them if there is another way. If they use a formula, ask them how they could find the volume if they forgot the formula.

Question B provides a visual reinforcement of the two methods mentioned in the Launch. Let students use cubes to build the prisms if needed.

- *What is a formula for finding the volume using Kurt's method?*
- *What is a formula for finding the volume using Natasha's method?*
- *Is there a relationship between the two formulas?*

For part (3), ensure that students are comfortable substituting values into their formulas.

- *If the length of a side of a cube is 15 centimeters, use each of the three formulas, Kurt's, Natasha's, and Dushane's, to compute the volume.*

For Question C, discuss the difference between half of a unit cube and a $\frac{1}{2}$-unit cube.

- *Are two $\frac{1}{2}$-unit cubes the same volume as a 1-unit cube? Why or why not?*
- *How many $\frac{1}{2}$-unit cubes fit inside a 1-unit cube?*

Question D encourages students to move from concrete situations to more abstract situations. Again, notice which formulas they tend to use.

- *What is being measured when you determine how much tape is needed?*
- *What is your process for finding how much tape is needed for each prism?*

Note that, unlike with surface area, if students add the perimeters of all the faces, they do not get the total edge length. They actually get twice the total edge length.

Summarize

Repeat some of the questions from Question B in the Explore.

- *What is a formula for finding the volume using Kurt's method?*
- *What is a formula for finding the volume using Natasha's method?*
- *How can you find the area of the base?*
- *Is there a relationship between the two formulas?*

Use one of the boxes from Problem 4.1 and ask different students to demonstrate Kurt's strategy and Natasha's strategy and explain why they work.

- *Dushane said that we should use the formula ℓ^3. Would this would work for rectangular prisms?*
- *How does having a fractional dimension affect how you find the volume of a rectangular prism?*

As a final check, you could hold up a juice or cereal box and ask students how they could find the volume.

ACE

Assignment Guide for Problem 4.2

Applications: 15–30 | Extensions: 56–65

Answers to Problem 4.2

A. **1.** Prism I: length = 5 cm, width = 4 cm, height = 1 cm; Prism II: length = 5 cm, width = 4 cm, height = 2 cm; Prism III: length = 5 cm, width = 4 cm, height = 5 cm

2. Prism I: volume = 20 cm^3
Prism II: volume = 40 cm^3
Prism III: volume = 100 cm^3

Students might have found the volume by multiplying length × width × height, or by finding the number of cubes in a layer and then multiplying the number of layers by the number of cubes in each layer.

3. Prism I: surface area $= 58$ cm^2
Prism II: surface area $= 76$ cm^2
Prism III: surface area $= 130$ cm^2

Students can use many methods. They can find the surface area by drawing a net and finding the area of the net, or by finding the area of the three faces they can see and doubling that area.

B. 1. Both strategies are correct. Since the base of a rectangular prism is a rectangle and its area can be found by multiplying length $\times$ width, the two methods are the same.

2. Kurt's: $V = \ell \times w \times h$, Natasha's: $V = B \times h$, where $V =$ volume, $\ell =$ length, $w =$ width, $h =$ height, and $B =$ area of base. The formulas are the same since $V = (\ell \times w) \times h = (B) \times h$.

Note that often an upper-case "B" is used to represent the area of a base, whereas a lower-case "b" is used to identify the base segment in a triangle or parallelogram.

3. 840 cm^3

4. The formula, volume $= \ell^3$, is the same as the other two formulas. The height, width, and length of a cube are the same, so they can be represented by the same letter or variable in the volume formula.

C. 1. Durian is correct that he cannot fit unit centimeter cubes exactly into the box, but he is incorrect in thinking that none of the formulas for volume work. Durian could find the correct volume by using a different sized cube to fill the box, or he could use the volume formula by multiplying the fraction edge lengths. Suppose the unit is a centimeter cube. Students can argue that they can either use parts of a 1-centimeter cube, or they can change the size of the unit cube to be $\frac{1}{4}$-centimeter cube or $\frac{1}{2}$-centimeter cube. This is an example of proportional reasoning, which is discussed in more depth in Grade 7 during *Stretching and Shrinking* and *Filling and Wrapping*.

2. 63.75 cm^3. If they use 1-centimeter cubes, the volume is 63.75 cm^3. If they use $\frac{1}{2}$-centimeter cubes, the volume is 8(63.75), or 510, $\frac{1}{2}$-centimeter cubes (which is equal to 63.75 cm^3 since there are eight $\frac{1}{2}$-centimeter cubes in one 1-centimeter cube). If they use $\frac{1}{4}$-centimeter cubes, the volume is 64(63.75), or 4,080, $\frac{1}{4}$-centimeter cubes (which is equal to 63.75 cm^3 since there are sixty-four $\frac{1}{4}$-centimeter cubes in one 1-centimeter cube).

D. 1. Prism I: volume $= 32$ in.^{3}
Prism II: volume $= 326.4$ cm^3
Prism III: volume $= 67.5$ in.^{3}

Methods will vary. One common method will be to multiply length, width, and height.

2. Prism I: surface area $= 64$ in.^{2}
Prism II: surface area $= 308.8$ cm^2
Prism III: surface area $= 133.5$ in.^{2}

Students may use different strategies for finding surface area. One common strategy is to find the area of the three unique faces of the rectangular prism, add those three faces' areas, then multiply that total by 2.

3. Prism I: tape $= 40$ in.
Prism II: tape $= 88.8$ cm
Prism III: tape $= 64$ in.

Find the perimeter of the base of the prism and multiply that perimeter by 2. This will be the amount of tape needed to cover the edges of the top and bottom bases. Then, multiply the height by 4. This will be the amount of tape needed to cover the vertical edges. Add two numbers (the doubled perimeter and the quadrupled height) together to find the total amount of tape needed to cover all the edges of the prism. Another way to find it is by recognizing that there are 4 edges with measure h units, 4 with measure ℓ units, and 4 with measure w units. Therefore, the amount of tape needed is $4(h + \ell + w)$.